Decorrelation estimates for random operators

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To fix ideas, let us take the random operator H_ω on $\ell^2(\mathbb{Z})$ defined by

$$\forall u \in \ell^2(\mathbb{Z}),\ [H_\omega u](n) = \omega_{n+1} u(n+1) + \omega_n u(n-1) \tag{1}$$

where $(\omega_i)_i$ are independent random variables, all uniformly distributed on $[1,2]$. Now, define $\Lambda_L = [-L, L] \cap \mathbb{Z}$ and let $H_\omega(\Lambda_L)$ be the restriction of $\chi_{\Lambda_L} H_\omega \chi_{\Lambda_L}$ to $\ell^2(\Lambda_L)$. $H_\omega(\Lambda_L)$ is a random matrix and has therefore $|\Lambda_L|$ eigenvalues, ordered increasingly

$$E_{1,\omega,L} \leq \cdots \leq E_{j,\omega,L} \leq \cdots \leq E_{|\Lambda_L|,\omega,L}.$$

The operator defined by (1) is ergodic, so there exists $\Sigma \subset \mathbb{R}$ such that $\mathrm{sp}(H_\omega) = \Sigma$ almost surely. Furthermore, there exists a function $N : \mathbb{R} \to \mathbb{R}$ strictly increasing, such that for almost every $E \in \mathbb{R}$ we have

$$N(E) = \lim_{L \to \infty} \frac{\sharp\{j, E_{j,\omega,L} \leq E\}}{|\Lambda_L|} \text{ p.s.}$$

The function N is the integrated density of states. Fix $E_0 \in \mathbb{R}$. We can now define the unfolded eigenvalues at E_0 by

$$\xi_{j,\omega,L,E_0} = |\Lambda_L| \left[N(E_{j,\omega,L}) - N(E_0) \right].$$

We study the local level statistics at E_0, i.e the limit of the following point process as $L \to \infty$

$$\Xi_{\omega,L,E_0}(\xi) = \sum_j \delta_{\xi_{j,\omega,L,E_0}}(\xi)$$

Following the work of [2], it suffices to prove that the operator defined in (1) satisfies three hypotheses, dynamical localization (see [2] for a precise result), the Wegner estimates and the Minami estimates, to prove that for all E_0 where these hypotheses are true and such that $N'(E_0) > 0$, Ξ_{ω,L,E_0} converges weakly to a Poisson process on $\mathbb{R}$ with intensity the Lebesgue measure.

The Wegner estimates are said to hold on a interval $\mathcal{I}$ if there exists $C > 0$ such that for all cube $\Lambda \subset \mathbb{Z}$ and all relatively compact interval $J \subset \mathcal{I}$, we have

$$\mathbb{E}\left(\mathbf{1}_J(H_\omega(\Lambda))\right) \leq C|J||\Lambda|$$

The Minami estimates ([4, 1]) are said to hold on a interval $\mathcal{K}$ if there exists $C > 0$ such that for all cube $\Lambda \subset \mathbb{Z}$ and all relatively compact interval $J \subset \mathcal{K}$, we have

$$\mathbb{E}\left[\mathbf{1}_J(H_\omega(\Lambda))\left(\mathbf{1}_J(H_\omega(\Lambda)) - 1\right)\right] \leq C|J|^2|\Lambda|^2$$

The lecture will be devoted to the study of the joint behavior of two local level statistics but taken at two different energy levels. In this context, we want to prove decorrelation estimates of distant eigenvalues, i.e a result that goes as follow : for $E_1 \neq E_2$, for L large enough and $\alpha \in (0,1)$, we have

$$\mathbb{P}\begin{pmatrix} sp(H_\omega(\Lambda_{L^\alpha})) \cap [E_1 - L^{-1}, E_1 + L^{-1}] \neq \emptyset \\ sp(H_\omega(\Lambda_{L^\alpha})) \cap [E_2 - L^{-1}, E_2 + L^{-1}] \neq \emptyset \end{pmatrix} = o\left(L^{\alpha-1}\right).$$

Such a result yields that if $N'(E_1) > 0$ and $N'(E_2) > 0$, then Ξ_{ω,L,E_1} and Ξ_{ω,L,E_2} converge weakly to two **independent** Poisson processes with intensity the Lebesgue measure.

We prove decorrelation estimates of eigenvalues, in the localized regime for several models, provided we have Wegner and Minami estimates. ([3, 5, 6]).

Références

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